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A Force/Position Control for Two-Arm Motion Coordination and Its Stability Robustness Analysis

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This paper presents a motion coordination of two robot manipulators coordinating an object. To coordinate the object, a force/position control scheme in a mode of leader/follower is devised. The dynamics of the object is incorporated into the dynamics of the leader arm, which yields a reduced order model of two arm system. In order to regulate interaction forces between two arms, the dynamics of the follower arm is expressed as force dynamic equations such that a novel direct force control scheme is devised. For the devised control scheme, a numerical simulation is shown. Under the coupling forces between two arms and two different type of bounded input disturbances, boundedness and asymptotic stability results based on a proposed Lyapunov function are shown. Also, a sufficient condition for a stability robustness is derived based on the Lyapunov approach.

Key Words: Force/Position Control, Direct Force Control, Boundedness, Asymptotic Stability, Stability Robustness, Lyapunov Approach

1. Introduction

One of the objectives in motion coordination of two interacting robot arms is to develop a control scheme such that many tasks in assembly, repair, and inspection that require multiple robot arms can be performed in a coordinated manner. A multitude of challenging research issues arise from multi-arm coordinated control e. g.(Nakano 1974 ; Luh 1987).

As one of control modes, a master/slave(leader/follower) mode of control is used for a cooperative motion coordination of two robot arms(Ahmad 1989 ; Ishida 1977 ; Ro 1989a ; Suh 1988 and Tarn 1986). In the research by Ishida(Ishida 1977), the master arm is controlled by a PID controller with a feedforward compensator, and the slave arm is controlled to follow the master arm by the force feedback from the wrist sensor. Another control method based on the master-slave control mode is shown by the linearlization(Tarn 1986). A dynamic coordina tor generates control action according to the relative position, velocity, and/or relative force errors between two arms where a linearlization scheme is applied. With additional degree-of-freedom allowed to the slave arm by changing its position, another master-slave mode of control is devised(Suh 1988). An application of master-slave mode of control to welding is shown with several other issues(Ahmad 1989). Another leader/follower control approach is presented with a reference model structure(Ro 1988a) where the leader arm is directed according to a prescribed reference model system while the follower arm follows via interacting force feedback.

Aside from the master/slave mode of control, a hybrid position/force control method is devised where the leader arm is coordinated to unconstrained directions while the force control is applied to constrained directions(Hayati 1986, Uchiyama 1988).

One of the important issues in the control of robot manipulators is stability. The small gain theorem(Desoer 1975) is used as a tool to test the stability and robustness of robot manipulator

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dynamics. One of the early applications of the theorem to robotic arms is shown(Spong 1984). In this paper, based on linearization of the robot dynamics with bounded model uncertainties, etc., the resulting error dynamics are guaranteed to be uniformly bounded. The previous work is extended to an n-link robotic manipulator using a multi-loop version of the small gain theorem(Spong 1985). Based on the small gain theorem, several generalized stability analyses, e. g., nonlinear time domain, frequency domain, etc. on the closed-loop robotic system is shown(Kazerooni 1987a). The work is extended to a two-arm robotic system where several generalized two-interacting robots are modeled, and their stability is studied(Kazerooni 1987b). Also, robustness conditions for the coupled dynamic system are derived and tested in depth for various types of bounded disturbances(Ro 1989b).

Another approach to the stability of robot manipulator dynamics is shown by the Lyapunov method. The stability of robotic systems with high nonlinearity and strong coupling between joints is addressed(Arimoto 1989), where the asymptotic stability of the system with a simplified PD or PID controller is guaranteed by the Lyapunov's Second method. In a two-arm system, a control law based on a stability condition via the Lyapunovs' Second method that has zero steady state errors is shown(Arimoto 1987). Aside from the robotic area, an approach to the convergence of certain second order differential equations is shown by a Lyapunov method(Ezeilo 1966). A global uniform ultimate boundedness stability result of a system based on the properties of a Lyapunov function is shown(Corless 1981). Also, an approach to show the asymptotic stability of a certain nonlinear retarded system is presented(Chukwu 1987).

As shown above, many control schemes relating to different issues are introduced in the area of two-arm robot control. Also, some studies of stability and robustness in robotic system are shown. In this paper, to our knowledge, the expression of the follower arm dynamics as a force dynamics and a formation of a direct force controller design are a novel approach. Also, a classification of two different type of input disturbances and a choice of a Lyapunov function are novel.

2. Two-Arm Dynamics

The dynamic equations for the coordination of a two-arm system are derived(Ro 1990) in which the object dynamics are treated, independently. In this paper, the dynamics of the object are incorporated into that of the leader arm and considered as a portion of the robot arm dynamics. Therefore, the $3n\times 1$ size of two-arm dynamic systems are reduced to a $2n\times 1$ size of dynamic systems. In this paper, for convenience, the leader arm is called arm b and the follower arm is called arm a. The equations of motion for the two arms coordinating an object can be expressed as the following :

$$H_a(q_a)\ddot{q}_a+C_a(q_a,\ \dot{q}_a)=\tau_a+J_a^T(q_a)F_a \quad (1)$$

$$H_b(q_b)\ddot{q}_b+C_b(q_b,\ \dot{q}_b)=\tau_b+J_b^T(q_b)F_b \quad (2)$$

where $q_a(q_b)$ is the $n\times 1$ joint angle vector for arm a(arm b) ; $\tau_a(\tau_b)$ is the $n\times 1$ joint torque vector for arm a(arm b) ; $H_a(H_b)$ is the $n\times n$ inertia matrix associated with arm a(arm b) ; C_a (C_b) is the nonlinear force vector of size $n\times 1$ including the gravity term ; and $J_a(J_b)$ is the $n\times n$ Jacobian matrix of arm a(arm b). $F_a(F_b)$ is an $n\times 1$ vector representing the forces and the moments at the point of interaction between arm a(arm b) and the object. Similar expressions for the dynamics of two-arm systems have been used by(Unseren 1990) and by others. The equations of motion for the object can be expressed as the following :

$$M_o\ddot{x}_o+Q_o(\dot{x}_o,\ x_o)=-L_a^TF_a-L_b^TF_b \quad (3)$$

where M_o is an $n\times n$ inertia matrix of an object ; Q_o is an $n\times 1$ nonlinear force vector ; and x_o is an $n\times 1$ vector representing the position and the orientation of the object center in the inertial space. The $n\times n$ matrix $L_a(L_b)$ represents the transformation matrix associated with a finite length between the center of the object and the interaction point a(b). The object is assumed to be rigidly grasped by arm b and there is no force sensor on arm b. The arm b and the object are kinematically related such that the velocity and acceleration of the mass center of the object is expressed in terms of the joint coordinates of robot arm b by the Jacobian matrix J_b as the following :

$$\dot{x}_o = L_b^{-1} x_b = L_b^{-1} J_b \dot{q}_b,$$
$$\ddot{x}_o = \dot{L}_b^{-1} J_b \dot{q}_b + L_b^{-1} \dot{J}_b \dot{q}_b + L_b^{-1} J_b \ddot{q}_b \quad (4)$$

where x_b is the velocity of the end effector of arm b in the task coordinate, and which is $\dot{x}_b = [v_b^T, w_b^T]^T$. v_b and ω_b are the 3×1 velocity vector and the 3×1 angular velocity vector of arm b. The object dynamics are expressed in the joint space with Eq. (4) as

$$M_o G(\dot{q}_b, \ddot{q}_b) + Q_{ob}(\dot{q}_b, q_b)$$
$$= -L_a^T F_a - L_b^T F_b \quad (5)$$

where

$$G(\dot{q}_b, \ddot{q}_b) = \dot{L}_b^{-T} J_b \dot{q}_b$$
$$+ L_b^{-T} \dot{J}_b \dot{q}_b + L_b^{-T} J_b \ddot{q}_b,$$

and Q_{ob} represents Q_o in the joint coordinates of arm b. Expressing Eq. (5) with regard to interacting force F_b and substituting it into Eq. (2) yield

$$F_b = -L_b^{-T} M_o G - L_b^{-T} Q_{ob} - L_b^{-T} L_a^T F_a \quad (6)$$

The dynamics of the object are incorporated into the dynamics of arm b by substituting Eq. (6) into Eq. (2).

$$H_b^* \ddot{q}_b = \tau_b + C_{ob} - \gamma F_a \quad (7)$$

where $H_b^* = H_b + J_b^T L_b^{-T} M_o L_b^{-T} J_b$, $\gamma = J_b^T L_b^{-T} L_a^T$, and

$$C_{ob} = -J_b^T L_b^T Q_{ob} - C_b - J_b^T L_b^{-T} M_o$$
$$\times (\dot{L}_b^{-1} J_b + L_b^{-T} \dot{J}_b) \dot{q}_b,$$

and H_b^* represents the inertia matrix of arm b dynamics incorporating that of the object ; C_{ob} represents the nonlinear force vector of arm b dynamics incorporating that of the object ; γ represents the Jacobian matrix reflecting the interacting force F_a in the joint space of arm a to arm b.

3. A Force/Position Controller Design for a Two-Arm Motion Coordination

3.1 A position control of leader arm via a computed torque method

A computed torque method is proposed to achieve position control of arm b. The controller τ_b for arm b dynamics is composed as

$$\tau_b = \hat{H}_b^* (q_{db} - K_{db} E - K_{pb} E)$$
$$- C_{ob} + D_b(t) \quad (8)$$

where $E = q_b - q_{db}(t)$: $q_{db}(t)$ is the desired joint angle vector ; $\hat{H}_b^*$ is an estimate of the inertia matrix of arm b ; C_{ob} is a feedforward estimate of the nonlinear force vector of arm b dynamics and the object dynamics ; $D_b(t)$ is a bounded input disturbance ; K_{db} and K_{pb} are $(n\times n)$ derivative and proportional gain matrix, respectively. Applying the control τ_b to Eq(7) yields

$$\ddot{q}_b = H_b^{*-1} \{ \hat{H}_b^* (\ddot{q}_{db} - K_{db} E - K_{pb} E)$$
$$- C_{ob} + D_b(t) + C_{ob} - \gamma F_a \} \quad (9)$$

Suppose that all the states and parameters of arm b are perfectly known such that $H_b^* = \hat{H}_b^*$ and $C_{ob} = C_{ob}$ are satisfied, then arranging Eq. (9) yields the following error dynamics as

$$\ddot{E} + K_{db} \dot{E} + K_{pb} E = -H_b^{*-1} \gamma F_a \quad (10)$$

where F_a represents the interacting force vector between arm a and arm b. In the error dynamic Eq. (10), the interacting forcing term $-H_b^{*-1} \gamma F_a$ disturbs the error dynamics. Therefore, by regulating the interacting forces, a desirable motion coordination is achieved. In order to regulate the interacting forces, the dynamics of arm a are expressed as force dynamic equations.

3.2 A force regulator for follower arm via a computed torque method

In order to express the arm a dynamics in terms of force, the arm a dynamics expressed in the joint space must be expressed in the task space beforehand. The joint space can be mapped to the task space by the Jacobian matrix as

$$\dot{q}_a = J_a^{-1} \dot{x}_a, \ \ddot{q}_a = \dot{J}_a^{-1} \dot{x}_a + J_a^{-1} \ddot{x}_a \quad (11)$$

where it is assumed that the Jacobian matrix is not singular such that there always exists an inverse Jacobian matrix. Substituting Eq. (11) into Eq. (1) and arranging it yields the following dynamic equations,

$$\ddot{x}_a = J_a H_a^{-1} (\tau_a + J_a^T F_a - C_a)$$
$$- J_a \dot{J}_a^{-1} \dot{x}_a \quad (12)$$

Eq. (12) represents the dynamics of arm a interacting with arm b where the acceleration and the velocity of the arm a dynamics are expressed in the task space. The gripper and force sensor are assumed not very stiff such that the interacting force can be expressed by the position differences of the end points between arm a and b in the task space as

$$F_a = K_{pp}(x_a - x_b) \tag{13}$$

where K_{pp} is an $n \times n$ diagonal stiffness gain matrix of sensor and gripper. Also, in Eq. (12), the velocity and acceleration of arm a in task space can be expressed by the first and the second derivative of force vector, respectively as

$$x_a = x_b + K_{pp}^{-1} F_a,\ \dot{x}_a = \dot{x}_b + K_{pp}^{-1} \dot{F}_a,$$
$$\ddot{x}_a = \ddot{x}_b + K_{pp}^{-1} \ddot{F}_a. \tag{14}$$

By substituting the above equations into Eq. (12) and arranging the equations, the force dynamic equations for arm a are expressed as

$$\begin{aligned}\ddot{F}_a =& K_{pp} J_a H_a^{-1}(\tau_a + J_a^T F_a - C_a) \\ & - K_{pp} J_a \dot{J}_a^{-1} K_{pp}^{-1} \dot{F}_a \\ & - K_{pp}(\ddot{x}_a + J_a \dot{J}_a^{-1} \dot{x}_b).\end{aligned} \tag{15}$$

In the open-loop Force dynamics in Eq. (15), there is a coupling term including the second derivative of x_b, which can be expressed in the joint space by the Jacobian relation as $\dot{x}_b = J_b \dot{q}_b$ and $\ddot{x}_b = \dot{J}_b \dot{q}_b + \dot{J}_b \ddot{q}_b$. In this way, the second derivative term $\ddot{x}_b$ can be expressed by the second derivative of the joint angles of arm b. With the assumption of $H_b^* = \hat{H}_b^*$ and $C_{ob} = \hat{C}_{ob}$, the second derivative of the joint angles in Eq. (15) is expressed in the first order states by the following relation $\ddot{q}_b = \ddot{q}_{db} - K_{db}\dot{E} - K_{pb}E + D_b(t) - \gamma F_a$. In the force dynamic equations of arm a, the interacting force F_a in the closed-loop arm b dynamics can be regulated by a direct force control scheme. A controller is designed as

$$\begin{aligned}\tau_a =& -(\dot{J}_a \hat{H}_a^{-1})^{-1} K_{pp}^{-1}\{(K_{pf} F_a \\ & + K_{df} \dot{F}_a) - J_a \dot{J}_a^{-1} K_{pp}^{-1} \dot{F}_a + K_{pp} J_b \gamma F_a\} \\ & - J_a^T F_a + \hat{C}_a + D_a(t)\end{aligned} \tag{16}$$

where K_{pf} and K_{df} are $n \times n$ proportional and derivative gain matrix, respectively; $\hat{C}_a$ is a feedforward compensating term of the nonlinear force vector of arm a dynamics; and $D_a(t)$ is a bounded input disturbance. Applying the control in Eq. (16) to Eq. (15) and arranging it with the assumption of $\hat{J}_a \hat{H}_a^{-1} = J_a H_a^{-1}$ and $\hat{C}_a = C_a$, then the closed-loop force dynamic equations of arm a which regulate the interacting force F_a are obtained as

$$\begin{aligned}& \ddot{F}_a + K_{da} \dot{F}_a + K_{pa} F_a \\ &= -K_{pp} J_b (\ddot{q}_{db} - K_{db} \dot{E} - K_{pb} E \\ & + P(q_a, q_b, \dot{q}_b) + D_a(t)).\end{aligned} \tag{17}$$

where $P(q_a, q_b, \dot{q}_b) = -(\dot{J}_b + J_a \dot{J}_a^{-1} J_b)\dot{q}_b$. For the follower arm, in order to regulate the interacting forces between two arms, a force control based on a computed torque method is used. This force control scheme is based on the information of the force and its derivative. The control scheme damps out the chattering which usually occurs when only a force feedback control is used. In control of the two-arm system, the gains for Eqs. (10) and (17) can be designed appropriately such that the interacting forces are regulated. In this way, the interacting forces are eventually regulated to zero in error dynamics. Therefore, arm b dynamics become independent of arm a dynamics such that a good motion coordination is achieved with dynamic decoupling. To validate the devised control scheme, a numerical example is shown by dual three degree-of-freedom planar arms.

4. Numerical Example

In this numerical example, the motion coordination of the two-arm system is simulated. The force/position control is applied to the system whose parameters are perfectly known. Dual three degree-of-freedom planar arms are used with a point mass at the center of each arm, which is shown in Fig. 1. The dimensions of the arms are

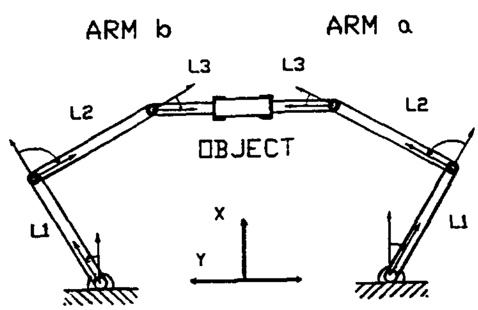

Fig. 1 Dual-arm system

Table 1 Dimension of dual-arm system

Arm, Obj.	Len. of arm	Mass	Inertia
L1	1(m)	2 kg	0.2 kg
L2	1(m)	2 kg	0.2 kg
L3	0.5(m)	1 kg	0.1 kg
Le	0.135(m)	0.5 kg	0.04 kg

also shown in Table 1. In the simulation of the force-position control scheme, the initial and the final position of the center of the object in the task space are given as: $x\_\{i\}=1.346$(m), $y\_\{i\}=0$, and $x\_\{f\}=1.8$(m), $y\_\{f\}=0$. For the desired input dynamics of arm b, a joint space scheme is applied with cubic polynomials. The computed torque method is applied to coordinate the dynamics of arm b to follow the dynamics of the desired system. The force control scheme based on a computed torque scheme is applied to regulate the interacting forces such that the dynamics of one arm is decoupled from that of the other. In control of the system, arm b is coordinated along the X-axis. Initial force excitations are 20 (Newton) along F_x and 25(Newton) along F_y. Also, the diagonal stiffness of the force sensor and gripper are 100(Newton/m). In the plots of the results of the numerical examples, the upper plots represent the regulation of the interacting forces and the lower plots represent the motion coordination of arm b in the task space.

In Fig. 2, the overdamped feedback gains of the controller are selected such that the initially excited interacting forces are not regulated to zero within a short time. The dynamics of arm b is affected by the interacting forces such that arm b cannot follow an input command within a short time. In Fig. 3, the control gains are critically designed such that the interacting forces are regulated to zero in a short time while the position of arm b is controlled to follow the desired input command in a short time. As a result, the dynamics of both arms is decoupled such that the controller shows a good motion coordination of the system. In Fig. 4, underdamped feedback gains of the controller are designed such that the initially excited interacting forces are regulated with a transient mode for a short period of time though it shows fast response. Also, the dynamics of arm b are affected by the transient mode of interacting forces, but the command following of arm b is achieved in a short time.

According to the numerical simulation results, it is shown that the proposed control schemes

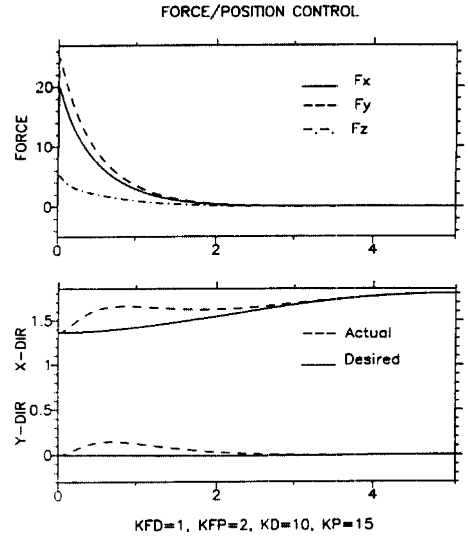

Fig. 2 Force/position control with overdamped gains

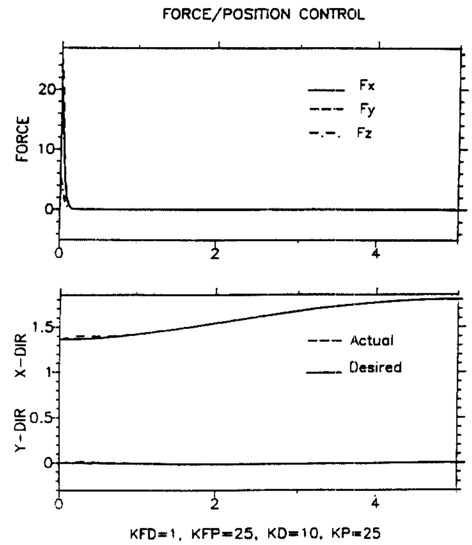

Fig. 3 Force/position control with critical-damped gains

exhibit a good motion coordination of the dual-arm system with appropriate control gains. As a result, the decoupling of the dynamics between arm a and arm b are achieved. However, with inappropriate feedback gains, the unstable system response is shown as in Fig. 5 such that an analysis of stability robustness is required.

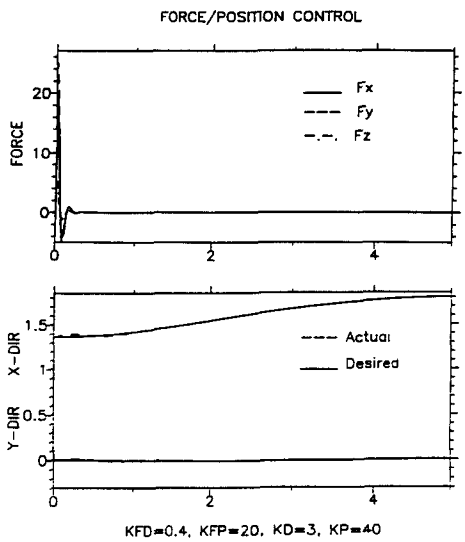

Fig. 4 Force/position control with underdamped gains

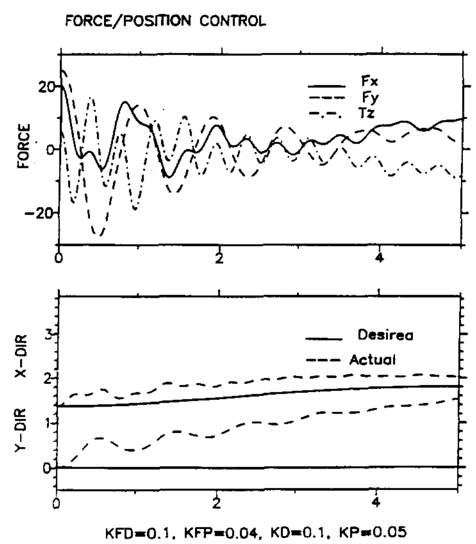

Fig. 5 Force/position control with inappropriate small gains

5. A Stability Analysis of the Closed-Loop Two-Arm Dynamics

The stability of the closed-loop two-arm dynamic system is not guaranteed due to the coupling forces. For this reason, general conditions to guarantee the stability of the closed-loop two-arm dynamic system are required. Rewriting the closed-loop two-arm dynamics in Eqs. (10) and (17) as $2n \times 1$ matrix equation yields

$$\ddot{X} + C\dot{X} + GX = Q(t, X, \dot{X}) \quad (18)$$

where

$$X = \begin{bmatrix} E \\ E_f \end{bmatrix}, \ C = \begin{bmatrix} K_{db} & 0 \\ 0 & K_{df} \end{bmatrix}, \ G = \begin{bmatrix} K_{pb} & 0 \\ 0 & K_{pf} \end{bmatrix},$$

$$Q = \begin{bmatrix} -\hat{H}_b^{*-1}(\gamma F_a - D_b(t)) \\ K_{pp}J_b(\ddot{q}_{db} - K_{db}\dot{E} - K_{pb}E + P + D_a(t)) \end{bmatrix}$$

and X is the $2n \times 1$ vector. C and G are the $2n \times 2n$ diagonal matrices. In the closed-loop two-arm system, there is a nonlinear coupling term $Q(t, X, \dot{X})$ between arm a and b. The dynamics of each arm are assumed to be affected by only the coupling forces instead of other forcing terms such as compensation errors or model uncertainties, etc.. Under this condition, a stability robustness analysis is shown for the linearized two-arm dynamics with nonlinear coupling forces.

5.1 Theorem of the stability of the closed-loop system

As an earlier work, an approach to the convergence of certain second order differential equation is shown by the Lyapunov method(Ezeilo 1966). Let E^{2n} denote the real Euclidean $2n$ dimensional space. Its Euclidean norm is denoted by $\|X\| = (X^T X)^{1/2}$. The problem here is to determine the conditions on the proportional and derivative feedback gains C and G with two conditions on the coupling forces Q such that asymptotic stability or boundedness of the coupled two-arm dynamics are guaranteed.

Suppose that

(i) the function $Q(t, X, Y)$ including coupled forces and input disturbances are upper bounded for $T - T_0 < \sigma$ as

$$\|Q(t, X, Y)\| \leq \lambda_2(\|X\|+\|Y\|) + \|Q(t, 0, 0)\| \quad (19)$$

uniformly in t where an appropriate $\lambda_2>0$ is constant; T_0 is the initial time, and σ is a sufficiently large time duration satisfying above boundedness.

(ii) the input disturbances are not consistent over sufficiently large time T such that the time dependent function is upper bounded as $\|Q(t, 0, 0)\| \leq \varepsilon_1$ in(i) for $0 \leq t \leq T_f < T$, where ε_1 is a finite constant, T_j is a finite time and T is a sufficiently large execution time.

(iii) the input disturbances are consistent over finite time T but upper bounded as $\|Q(t, 0, 0) \leq Q_{max}$ for $0 \leq t \leq T$, where Q_{max} is a finite constant.

THEOREM 1.

With assumption (i) and (ii), $\|X\| \to 0$ and $\|Y\| \to 0$ for all sufficiently large t.

THEOREM 2.

With assumption (i) and (iii), $\|X\|$ and $\|Y\|$ are upper bounded with respect to a ball centered at $2n$ and $2n$ dimensional spaces, respectively.

In (ii), the square norm of $Q(t, 0, 0)$ is upper bounded such that $\int_0^T \|Q(t, 0, 0)\|^2 dt < \infty$. In theorem 1. and 2., there exists a constant $\Lambda_2>0$ whose magnitude depends only on λ_2, C, and G such that if $\lambda_2 \leq \Lambda_2$, asymptotic stability or boundedness of X and Y are obtained.

5.2 Proof of the theorem 1.

For the proof of the asymptotic stability, Eq. (18) can be expressed as

$$\dot{X} = Y,$$
$$\dot{Y} = -CY - GX + Q(t, X\dot{X}) \quad (20)$$

Also, a Lyapunov function $V=V(X, Y)$ is chosen as

$$2V = \langle Y+CX, Y+CX\rangle + \langle Y, Y\rangle + 2\langle GX, X\rangle \quad (21)$$

Here the Lyapunov function $V(X, Y)$ is a convex function and positive scalar because the matrix G is defined as positive definite. In addition to the positive quantity of the Lyapunov, an estimate for $\dot{V} \equiv d/dt V(X(t), Y(t))$ corresponding to any solution (X, Y) of Eq. (20) is required. For convenience, $Q(t, x, \dot{x})$ is expressed as $Q(.)$ and $Q(t, 0, 0)$ is expressed as $Q'(.)$. Differentiating the Lyapunov function of Eq. (21) yields

$$\begin{aligned}\dot{V} &= \langle Y+CX, -CY-GX+Q(.) + CY\rangle \\ &\quad + \langle Y, -CY-GX+Q(.)\rangle \\ &\quad + 2\langle GX, Y\rangle \\ &= \langle CX, GX\rangle - \langle Y, CY\rangle \\ &\quad + \langle 2Y+CX, Q(.)\rangle \end{aligned} \quad (22)$$

The estimate of the first term in the right hand side of Eq. (22) is

$$\begin{aligned}&\langle CX, GX\rangle \\ &= \langle CGX, X\rangle \geq \lambda_c\lambda_g\|X\|^2 \\ &= \lambda_{cg}\|X\|^2\end{aligned} \quad (23)$$

where λ_c and λ_g are the least eigenvalues of matrix C and G respectively. Also, $\lambda_{cg} \equiv \lambda_c\lambda_g > 0$ because C and G are defined as diagonal positive definite matrices. In a similar way, the second term of Eq. (22) is estimated as

$$\langle Y, CY\rangle = \langle CY, Y\rangle \geq \lambda_c\|Y\|^2 \quad (24)$$

Applying Schwarz's inequality to estimate the remaining term in the expression $\dot{V}$ in Eq. (22) yields

$$\begin{aligned}&|\langle 2Y+CX, Q(.)| \\ &\leq \lambda_3(\|X\|+\|Y\|)Q(.)\|\end{aligned} \quad (25)$$

for some constant $\lambda_3>0$ whose magnitudes can be estimated with (19) as

$$\begin{aligned}&|\langle 2Y+CX, Q(.)| \\ &\leq \lambda_3(\|X\|+\|Y\|)Q(.)\| \\ &\leq \lambda_3(\|X\|+\|Y\|)\{\lambda_2(\|X\|+\|Y\|)+\|^2 Q'\|\} \\ &\leq 2\lambda_2\lambda_3(\|X\|^2+\|Y\|^2) \\ &\quad + \lambda_4(\|X\|^2+\|Y\|^2)^{1/2}\end{aligned} \quad (26)$$

where λ_{mc} is the largest eigenvalue of C matrix such that $\lambda_3 = \max(2, \lambda_{mc})$. With the above estimates, the inequality can be expressed with the assumption of the boundedness of the disturbance term $Q(t, X, Y)$ in Eq. (20). In Eq. (26), $\lambda_4 = 2^{1/2}\lambda_3\|Q'\|$. Thus, the inequality is achieved by putting all the estimates of the various terms in Eqs. (24, 25, 26) into the expression for $\dot{V}$ in Eq. (22) as

$$\begin{aligned}\dot{V} &\leq -(\lambda_{cg} - 2\lambda_2\lambda_3)\|X\|^2 \\ &\quad -(\lambda_c - 2\lambda_2\lambda_3)\|Y\|^2 \\ &\quad + \lambda_4(\|X\|^2+\|Y^2)^{1/2}.\end{aligned} \quad (27)$$

Hence, if λ_2 is defined as

$$\lambda_2 \leq \Lambda_2 = \frac{1}{2}\min(\lambda_{cg}\lambda_3^{-1}, \lambda_c\lambda_3^{-1}) = \frac{1}{2}\min(\lambda_g, 1)\lambda_c\lambda_3^{-1}$$

which is a sufficient condition for $\dot{V}$ to become negative semi-definite which is shown by the following procedures. In Eq. (27), with the sufficient condition,

$$\dot{V} \leq -2\lambda_5(\|X\|^2+\|Y\|^2) + \lambda_4(\|X\|^2+\|Y\|^2)^{1/2} \quad (28)$$

where $\lambda_5=\min(\lambda_{cg}-2\lambda_2, \lambda_3\lambda_c-\lambda_2\lambda_3)$. In order for $\dot{V}\leq 0$, the second term of Eq. (27) has to be smaller than the first term. To see this, we go back to Eq. (21) where the first term in the righthand side can be estimated as

$$\lambda_{11}(\|X\|^2+\|Y\|^2) \leq \langle Y+CX, Y+CX\rangle \leq \lambda_{12}(\|X\|^2+\|Y\|^2)$$

where λ_{11} and λ_{12} are positive and are related to the C matrix. In a similar way, the rest of the terms can be estimated as

$$\lambda_{13}(\|X\|^2+\|Y\|^2) \leq \langle Y, Y\rangle + 2\langle GX, X\rangle \leq \lambda_{14}(\|X\|^2+\|Y\|^2)$$

where λ_{13} and λ_{14} are positive and are related to the G matrix. From the above two inequalities, the following relationship is obtained :

$$\alpha_1(\|X\|^2+\|Y\|^2) \leq V \leq \alpha_2(\|X\|^2+\|Y\|^2) \quad (29)$$

where $\alpha_1=\lambda_{11}+\lambda_{13}$ and $\alpha_2=\lambda_{12}+\lambda_{14}+\lambda_{15}$. Rewriting Eq. (28) with Eq. (29) yields

$$\dot{V} \leq -2(\lambda_5/\alpha_2)V + (\lambda_4/\alpha_1)V^{1/2} \quad (30)$$

Let $D\equiv 2^{1/2}(\lambda_3/\alpha_1)$ and $\gamma_2\equiv\lambda_5/\alpha_2$; recall that $\lambda_4 = 2^{1/2}\lambda_3(\|Q(t, 0, 0)\|)$, then the above equation can be expressed as

$$\dot{V}+\gamma_2 V \leq -\gamma_2 V + D\|Q'(.)\| V^{1/2} \quad (31)$$

Eq. (31) can be expressed as

$$\dot{V}+\gamma_2 V \leq M \quad (32)$$

where $M=V^{(1/2)}(D\|Q'(.)\|-\gamma_2 V^{1/2})$. From the definition of M, the following relationship can be obtained : $M\leq D_1\|Q'(.)\|^2$, for all $t\geq\xi$, $D_1=D_1(D, \gamma_2)>0$ is constant, which is proved by the following process : If Q and V are such that $D\|Q'(.)\|<\gamma_2 V^{1/2}$, then $M\leq 0$. which yields $\dot{V}\leq 0$; and if $D\|Q'(.)\|>\gamma_2 V^{1/2}$ then, with the previous equation, $M\leq V^{(1/2)}D\|Q'(.)\| = D^2\|Q'\|(.)\|^2/\gamma_2$ such that $D_1=D^2/\gamma_2$. Rewriting (32) yields

$$\dot{V}+\gamma_2 V \leq D_1\|Q'(.)\|^2 \quad (33)$$

Solving the above inequality for V yields

$$V(t) \leq e^{-\gamma_2 t}[e^{\gamma_2\xi}V(\xi) + D_1\int_\xi^t \|Q(s, 0, 0)\|^2 e^{\gamma_2 s}ds] \quad (34)$$

Rewriting the above inequality by the mean-value theorem for integrals with $\xi<t_1<t$ yields

$$\|X\|^2+\|Y\|^2 \leq (1/\alpha_1)[e^{-\gamma_2 t}A_1 + D_1 e^{-\gamma_2(t-t_1)}\int_\xi^t \|Q(s, 0, 0)\|^2 ds].$$

where $A_1=e^{\gamma_2\xi}V(\xi)$. With the stated condition(ii) in the theorem,

$$\int_\xi^\infty \|Q(t, 0, 0)\|^2 dt < \infty$$

the above inequality satisfies $\|X\|^2+\|Y\|^2\to 0$ as $t\to\infty$ which requires $\|X\|\to 0$ and $\|Y\|\to 0$ as $t\to\infty$ E. O. F

5.3 Proof of the theorem 2

With the assumption (i) and (iii), the consistent time dependent disturbances are upper bounded such that $\|Q(t, 0, 0)\|<Q_{max}$. Note that Q_{max} is a finite constant. In Eq. (28),

$$\dot{V}(X, Y) \leq -2\lambda_5(\|X\|^2+\|Y\|^2) + 2^{1/2}\lambda_3 Q_{max}(\|X\|^2+\|Y\|^2)^{1/2} \leq -2\lambda_5 Z + 2^{1/2}\lambda_3 Q_{max} Z \quad (35)$$

where $Z^2=\|X\|^2+\|Y\|^2$. If $Z<\lambda_3 Q_{max}/(2^{1/2}\lambda_5)$ which is $\|X\|^2+\|Y\|^2<(\lambda_3 Q_{max}/(2^{1/2}\lambda_5))^2$, $\dot{V}$ is positive, then, $V(X, Y)$ increases such that X and Y increases. At a certain region, $Z>\lambda_3 Q_{max}/(2^{1/2}\lambda_5)$ becomes such that $\dot{V}$ is nonpositive and $V(X, Y)$ decreases. The repeating process implies that X and Y bounded by finite constants. E.O.F

6. Conclusions

A reduced order model of the two-arm system dynamics is derived such that the dynamics becomes computationally feasible. Follower arm dynamics is expressed as a novel force dynamic equations which are appropriate for a direct force control with proportional and derivative gains.

By the devised direct force controller, the interacting forces between two arms are regulated to zero. In this way, the dynamics of the two-arm system can be decoupled. Also, for the closed-loop two-arm system with two different type of input disturbances, a sufficient condition is derived for the asymptotic stability and boundedness based on a proposed Lyapunov function. If the Euclidean norm of the coupled disturbing forces of the system is upper-bounded by the norm of the states weighted by derivative and proportional gains, the stability of the system is guaranteed. Numerical examples show the validatity of the devised controller.

References

Ahmad, A. and Luo, S., 1988, "Coordinated Motion Control of Multiple Robotic Devices for Welding and Redundancy Coordination through Constrained Optimization in Cartesian Space," IEEE J. of Robotics and Automation, Vol. RA-5, No. 4, pp. 409～417.

Arimoto, S., Miyazaki, F. and Kawamura, S., 1987, "Cooperative Motion Control of Multiple Robot Arms or Fingers," Proc. of IEEE Int. Conf. on Robotics and Automation, pp. 1407～1412.

Arimoto, S., Miyazaki, F., Lee, H. G. and Kawamura, S., 1989, "Rivival of Lyapunov's Direct Method in robot Control and design," Proc. of American Control Conf., pp. 1764～1769.

Craig, J. J., Hsu, P. and Sastry, S., 1986, "Adaptive Control of Mechanical Manipulators," IEEE Int. Conf. Robotics and Automation, San Francisco, CA.

Cai, L. and Goldenberg, A. A., 1988, "Robust Control of Unconstrained Maneuver and Collision for a Robot Manipulator with Bounded Parameter Uncertainty," Proc. IEEE Int. Conf. on Robotics and Automation, pp. 1010～1015.

Chen, Y. H., 1986, "On the Deterministic Performance of Uncertain Dynamical Systems," Int. J. of Control, Vol. 43. No. 5, pp. 1557～1579.

Chen, Y. H., 1991, "Robust Computed Torque Scheme for Mechanical Manipulators : Nonadaptive Versus Adaptive," J. of Dynamic Systems, Measurement, and Control, Vol. 113, pp. 324～327.

Chukwu, E. N., 1987, "Global Null Controllability of Nonlinear Delay Equation with Controls in Compact Set.," Journal of Optimization Theory and Application, Vol 53, No. 1, pp. 43～57.

Corless, M. and Leitmann, G., 1981, "Continuous State Feedback Guaranteeing uniform Ultimate Boundedness for Uncertain Dynamic Systems," J. Automatic Control AC-26 : pp. 1139～1143.

Desoer, C. A. and Vidyasagar, M., 1975, "Feedback Systems : Input-Output Properties," Academic Press, New York.

Ezeilo, J. O. C., 1966, "On the Convergence of Solutions of Certain Systems of Second Order Differential Equations," Ann. Mat. Pura Appl.(4) 72, pp. 239～252.

Hayati, S., 1986, "Hybrid Position/Force Control of Multi-Arm Cooperating Robots," Proc. IEEE Int. Conf. on Robotics and Automation, pp. 82～89.

Ishida, S. T., 1977, "Force Control in Coordination of Two Arms," Proc. of the 5th Int. Joint Conference on Artificial Intelligence, pp. 717～722.

Kazerooni, H., 1987a, "On the Stability of the Robot Compliant Motion Control(Input Output Approach)," Proc. of 26th Conf. on Decision and Control, Los Angeles, CA, Dec., pp. 222～232.

Kazerooni, H., 1987b, "Stability Analysis of Two Cooperating Robot Manipulators," In Modeling and Control of Robotic Manipulators and Manufacturing Processes, ASME, New York, pp. 203～212.

Luh, J. Y. S. and Zheng, Y. F., 1987, "Constrained Relations between Two Coordinated Industrial Robots for Motion Control," Int. J. Robotics Research, Vol. 6., No. 3, pp. 60～70.

Nakano, N., Ozaki, S., Ishida, T. and Kata, I., 1974, "Cooperational Control of the Anthropomorphous Manipulator MELANM," Proc. of Fourth Int'l Conf. on Architectural Robots, pp. 251～260.

Ro, P. I., Choi, H. S and Lee, G., 1990, "A

Dynamic Decoupling Control Scheme for Two-Arm Coordination via State Feedback," Int. Simposium on Robotics and Manufacturing, pp. 853~858.

Ro, P. I. Ro. and Choi, H. S., 1991, "Force-position Control and Position-Position Control for Two-Arm Coordination with Dynamic Decoupling," Proc. of ASME WAM91, pp. 109~114.

Ro, P. and Youcef-Toumi, K., 1989a, " A Reference Model Following Control for Motion Coordination of Two Dynamically Interacting Systems," Proc. of ASME WAM89, San Francisco, Dec.

Ro, P. I. and Youcef-Toumi, K., 1989b, "Robust Motion Coordination of Two Dynamically Interacting Systems with Uncertainties and Bounded Disturbances," 28th IEEE Conference on Decision and Control, Tampa.

Spong, M. W. and Thorp, J. S. and Kleinwakes, J. M., 1984, "The Control of Robot Manipulators with Bounded Input Part II : Robustness and Disturbance Rejection,"Proc. of 23rd Conf. on Decision and Control, Las Vegas, NV.

Spong, M. W. and Vidyasagar, M., 1985, "Robust Linear Compensator Design for Nonlinear Robotic Control," Proc. of IEEE Int. Conf. on Robotics and Automation, pp. 954~959.

Suh, I. H. and Shin, K. G., 1988, "Coordination of Dual Robot Arms Using Kinematic Redundancy," In Proceedings of 1988 International Conference on Robotics and Automation, Philadelphia, PA, April, pp. 504~509.

Tarn, T. J. and Bejczy, A. K. and Yun, X., 1986, "Coordinated Control of Two Robot Arms," Proc. of IEEE Int. Conf. on Robotics and Automation, San Francisco, pp. 1193~1202.

Uchiyama, M. and Dauches, P., 1988, "A Symmetric Hybrid Position/Force Control Scheme for the Coordination of Two Robots," Proc. IEEE Int. Conf. on Robotics and Automation, pp. 350~356.